Math 121 Test #3 Answers Fall 2009

$(46, 57, 74, 88, 95)$, $\overline{x} = 73.2$.

1. (15 pts) $(4, 11, 12, 14, 15)$, $\overline{x} = 11.4$.

   There is only one sample in this problem and the statistic clearly involves proportions, not means. Therefore, this is a one-proportion z test. The seven steps are

   1. Let p be the proportion of likely voters who approve of President Obama's performance.
   $H_0 : p = 0.50$.
   $H_1 : p < 0.50$.

   2. $\alpha = 0.01$.

   3. $z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$.

   4. We are given that $\hat{p} = 0.46$. So $z = \dfrac{0.46 - 0.50}{\sqrt{\frac{(0.50)(0.50)}{1500}}} = -\dfrac{0.04}{0.0129} = -3.098$.

   5. p-value $=$ `normalcdf(-E99,-3.098)` $= 9.730 \times 10^{-4}$.

   6. Reject H_0.

   7. President Obama's approval rate is less than 50%.

   You can use `1-PropZTest` on the TI-83 to get the values in steps 4 and 5.

2. (10 pts) $(0, 6, 8, 10, 10)$, $\overline{x} = 7.4$.

   (a) The formula is
   $$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$
   For a 95% confidence interval, $\alpha = 0.05$ and $z_{0.025} = 1.960$. The confidence interval is
   $$0.46 \pm 1.96\sqrt{\frac{(0.46)(0.54)}{1500}} = 0.46 \pm 0.0252.$$
   You could also use the `1-PropZInt` function in the TI-83. It produces the answer `(.43478,.48522)`, which is equivalent to the previous answer.

   (b) The margin of error is 0.0252. If you used the TI-83 in part (a), then find the margin of error as $0.48522 - 0.46 = 0.02522$.

3. (15 pts) $(6, 9, 12, 14, 15)$, $\overline{x} = 11.9$.

   This is a two-sample hypothesis-testing problem concerning proportions. The seven steps are

1. Let p_1 be the strong-disapproval rate on Nov. 5 and let p_2 be the strong-disapproval rate on Nov. 19.
$H_0 : p_1 = p_2$.
$H_1 : p_1 < p_2$.

2. $\alpha = 0.05$.

3. $z = \dfrac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$, where $\hat{p}$ is the pooled estimate for the common value of p_1 and p_2.

4. We are given that $\hat{p}_1 = 0.38$ and $\hat{p}_2 = 0.41$. That represents 570 responses in the Nov. 5 sample and 615 responses in the Nov. 19 sample. The pooled estimate $\hat{p}$ is $\dfrac{570 + 615}{1500 + 1500} = \dfrac{1185}{3000} = 0.395$. So

$$z = \frac{(0.38 - 0.41) - 0}{\sqrt{(0.395)(0.605)\left(\frac{1}{1500} + \frac{1}{1500}\right)}} = \frac{-0.03}{0.01785} = -1.681.$$

5. p-value = `normalcdf(-E99,-1.681)` $= 0.0464$.

6. Reject H_0.

7. The strong-disapproval rate increased from Nov. 5 to Nov. 19.

You can use `2-PropZTest` on the TI-83 to get the values in steps 4 and 5.

4. (12 pts) $(0, 7, 10, 12, 12)$, $\overline{x} = 8.3$.

(a) $P(-4 < t_1 < 4) =$ `tcdf(-4,4,1)` $= 0.8440$.

(b) $P(t_{10} > 1.5) =$ `tcdf(1.5,E99,10)` $= 0.0823$.

(c) $P(t_{20} > 1.5) =$ `tcdf(1.5,E99,20)` $= 0.0746$.

5. (12 pts) $(9, 9, 12, 12, 12)$, $\overline{x} = 10.6$.

(a) We should use z. The sample size is large, but more importantly, we never use t for proportions.

(b) The statistic is the sample mean, the sample size is small, σ is unknown, and the data appear to come from a normal population. Therefore, we should use t.

(c) In this case, we must use z. We use t only when σ is unknown (and the population is normal).

(d) We could use either z or t. The better choice is t, but because of the large sample size, it is all right to use z.

6. (15 pts) $(0, 8, 10, 11, 15)$, $\overline{x} = 8.9$.

   I thought it was clear that the average of was 4.17 million acres *per year*. Maybe that wasn't clear. I probably should have stated it. Conceivably, one could talk about the number acres *per wildfire*, but then, to work the problem, you would have to know the number of wildfires. That was not given in the problem, so you would have to rule out that interpretation.

   Therefore, there are 49 observations in the sample, one for each year. The sample average is 4.17 (million acres) and the question is, does that support the claim that the true average is less than 5 million. This problem concerns means and involves one sample. Therefore, it is a one-sample hypothesis-testing problem for means. The seven steps are

   1. Let μ be the average number of acres (in millions) destroyed in wildfires each year.
   $H_0 : \mu = 5$.
   $H_1 : \mu < 5$.

   2. $\alpha = 0.05$.

   3. Because the sample size is large, you may use z. There is no mention of the population being normal, so it would be best not to use t, although I would allow it. The test statistic is
   $$z = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}.$$

   4. $z = \dfrac{4.17 - 5}{2.15/\sqrt{49}} = \dfrac{-0.83}{0.0371} = -2.702.$

   5. p-value $=$ `normalcdf(-E99,-2.702)` $= 0.00344$. If you said t in step 3, then you should calculate p-value $=$ `tcdf(-E99,-2.702,48)` $= 0.00475$. (Not enough difference to matter.)

   6. Reject H_0.

   7. The average number of acres destroyed in wildfires is less than 5 million per year.

   You can use `Z-Test` (or `T-Test`) on the TI-83 to get the values in steps 4 and 5.

7. (6 pts) $(0, 1, 2, 6, 6)$, $\overline{x} = 3.2$.

   The formula is $\overline{x} \pm z_{\alpha/2}\left(\dfrac{s}{\sqrt{n}}\right)$. For 90% confidence, $\alpha = 0.10$ and $z_{0.05} = 1.645$. Substitute the values and get
   $$4.17 \pm 1.645\left(\frac{2.15}{\sqrt{49}}\right) = 4.17 \pm 0.505.$$

   You can use the function `ZInterval` on the TI-83 to get the answer. The TI-83 gives `(3.6648,4.6752)`. If you used TInterval, then you got `(3.6549,4.6851)`.

8. (15 pts) $(5, 9, 13, 14, 15)$, $\overline{x} = 11.5$.

The problem concerns means and there are two samples. The seven steps are

1. Let μ_1 be the average electric bill before installment and let μ_2 be the average electric bill after installment.
$H_0 : \mu_1 = \mu_2$.
$H_1 : \mu_1 > \mu_2$.

2. $\alpha = 0.05$.

3. The sample sizes are small and σ_1 and σ_2 are unknown, but the populations appear to be normal, so we must use t. Also, s_1 and s_2 have similar values, so we may use the pooled estimate s_p for the common value σ. (It is all right if you don't do that.) The test statistic is

$$t = \frac{(\overline{x}_1 - \overline{x}_2) - 0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

where s_p is the pooled estimate

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}.$$

4. The pooled estimate is $s_p = \sqrt{\frac{(11)(12)^2 + (11)(10)^2}{22}} = 11.045$. Then

$$t = \frac{(135 - 127) - 0}{11.045\sqrt{\frac{1}{12} + \frac{1}{12}}} = \frac{8}{4.509} = 1.774.$$

5. p-value $=$ `tcdf(1.774,E99,22)` $= 0.0450$.

6. Accept H_0.

7. The average electric bills before and after installment of the new windows are the same.

You can use `2-SampTTest` on the TI-83 to get the values in steps 4 and 5.